Time-Periodic Control of a Multiblade Helicopter

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The equations of motion for a rigid helicopter containing four blades free to flap and lag are determined. Control techniques are developed which stabilize the entire system for a variety of flight conditions. A modal control technique, based on Floquet theory, is used to eliminate multiple-blade instabilities by controlling pairs of unstable roots at a specific design point. Another modal controller is designed for the resulting new system which shifts a second pair of unstable roots to desired locations. This process is repeated until all instabilities are eliminated. Numerical inaccuracies, however, limit the number of possible repetitions of this procedure.

Introduction

A HELICOPTER is a mechanically complex aircraft whose stability and control characteristics are often marginal unless a reliable automatic control system is used. Effective modeling of dynamic effects is crucial to the safe design of a successful helicopter. It is, therefore, essential to consider the basic dynamic behavior of helicopters so that potential instabilities can be simulated and eliminated in the design process.

To study the stability characteristics of a coupled rotor/fuselage system, an accurate mathematical model is required. Several authors[1-3] have developed the equations of motion of the entire helicopter, including both the rotors and the airframe. Bousman[4] obtained experimental data on the stability of a hingeless rotor mounted on a special gimbaled support, which simulated body pitch and roll degrees of freedom. He then compared this data with theoretical results he had obtained from an analytical model. Other studies[5-14] have introduced analytical models to examine coupled rotor/fuselage dynamics. Furthermore, a detailed description of the procedure necessary to obtain a set of rotor/fuselage equations of motion was accomplished by Venkatesan and Friedmann.[15] These authors presented a set of governing coupled differential equations for a model representing a multiple rotor, hybrid heavy lift helicopter. They subsequently reduced these equations to a single rotor model and obtained a simplified system of coupled rotor/fuselage equations, which produced results agreeing favorably with experimental data.[16] The model included a rotor with three or more hinged, rigid blades and, in deriving the system's equations of motion, neglected terms second order and higher.

The stability of a coupled rotor/fuselage system can be determined in a a variety of ways. Because of the complexity of the equations, a number of complex analyses have been developed and implemented in sophisticated computer programs.[11,12,17] It is possible, however, to simplify the coupled rotor/fuselage equations of motion so that relatively uncomplicated procedures can be used to determine the system's stability.[7,8,10,13] For example, Straub and Warmbrodt[7] and Straub[13] converted linearized perturbation equations with periodic coefficients into a system with constant coefficients using a Fourier coordinate transformation and determined system stability using a standard eigenvalue analysis.

Rather than just documenting where the instabilities of an isolated rotor blade or a coupled rotor/fuselage system occur, it is desirable to actually reduced or eliminate those instabilities. Miyajima[5] developed a stability and control augmentation system whereby the helicopter was represented by a six-degree-of-freedom rigid body (excluding the rotors) with constant factors used as weightings for the state and control variables. A least-squares design method was applied to determine the control augmentation system. Then, blade-flapping motion was included using the method of multiblade coordinates, and linear optimal control theory was applied to determine the appropriate feedback gains for the stability augmentation system.

Straub and Warmbrodt,[7] after approximating the periodic linearized coupled rotor/fuselage equations of motion with constant coefficients, used state-variable feedback with appropriate closed-loop feedback phase and gain margins. In addition, they used active blade pitch control with a conventional swashplate to increase helicopter rotor/fuselage damping. They obtained increased damping levels and eliminated ground resonance instabilities for a wide variety of rotor configurations. Straub[13] extended these results by applying multivariable optimal control techniques to control aeromechanical stability at all rotor speeds.

The feedback controllers described above typically adjusted damping levels to delay or eliminate instabilities. In addition, the equations of motion were typically approximated by a set of linear ordinary differential equations with constant coefficients.

Recently, active control of helicopter blade flapping has been accomplished using Floquet theory to allow pole placement in linear periodic systems.[18,19] This method works directly with the time-periodic linear equations and alters the unstable eigenvalues of the periodic system while leaving the others unchanged. Calico and March[18] applied this modal control technique to control the flapping motion of a single helicopter blade by using a flap torque actuator at the blade root. Subsequently, Calico and Wiesel[19] implemented a pole-placement control scheme by using the collective and cyclic pitch controls on a conventional swashplate. Furthermore, they extended their analysis to include two blades and examined the blades' flapping stability throughout the flight regime. Their controller eliminated the flapping instabilities of one and two helicopter blades at high advance ratios. Although both studies[18,19] dealt only with blade-flapping instabilities at

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high advance ratios, the technique utilized is directly applicable to controlling the instabilities of more complex helicopter models.

This research effort extends the modal control technique to a much more realistic helicopter model. Specifically, the current model includes blade flap and lag motions for a four-bladed helicopter and the rigid-body motion of the helicopter in the longitudinal-vertical plane. The modal control technique has the advantage, over previous control attempts, to reduce or eliminate rotor/fuselage[7,13,20] instabilities, of working directly with the system's linear time-periodic equations of motion.

The scope of this paper is limited to examining the effects of the modal control technique on a more complex mathematical model. As such, it is not intended to be a practical concept since feedback of the large state vectors considered is easily postulated but impractical for actual implementation. Furthermore, controllability and observability of the helicopter are not addressed. It is possible that numerical anomalies are a result of a lack of controllability and observability of some of the Floquet characteristic values considered. The importance of this issue requires more consideration than the scope of this paper allows. Similarly, actual control of the helicopter, whether through cyclic or collective pitch mechanisms, is not specifically addressed. Again, the intent of the paper is not to conduct an analysis as to why, for example, collective pitch is effective in placing some of the poles and ineffective in placing others. Rather, this paper is to demonstrate the effectiveness of the modal control technique. Finally, this technique is not compared to linear time-invariant (LTI) or linear quadratic regulator (LQR) techniques; that is a topic of a future paper.

Dynamics

A helicopter in free flight with fixed rotors has six rigid-body degrees of freedom. Additional degrees of freedom are introduced when the rotors dynamics are included. For a rotor with rigid hinged blades, there are three additional degrees of freedom for each blade, namely, flap, lag, and pitch. For this study, pitch is used for control, and flap and lag are free variables. The flap-lag equations of motion of an isolated rotor blade have been developed in an earlier paper.[21] The helicopter considered possesses a single main rotor, which is assumed to be directly over the helicopter's center of mass and provides lift; propulsive force; and roll, pitch, and vertical control for the helicopter. In addition, a horizontal tail is provided to add static stability with respect to the angle of attack.

The Earth is assumed to be flat and nonrotating, and perturbations about a straight and level flight equilibrium are examined. Parasitic drag on the main rotor and side forces on the fuselage are neglected. It is assumed that the sideslip angle, as well as the horizontal tail's incidence, is zero. The tail's downwash is a small angle and can be neglected; hence, the induced velocity from the horizontal tail is also neglected. The acceleration of gravity is constant and the properties of the atmosphere are known functions of altitude.

The tilt of the main rotor's thrust vector with the tip path plane and the tilt due to the helicopter's angular velocity must be considered in the stability analysis.[2] Furthermore, coupling terms between the helicopter's rigid-body motion and the blades' flap and lag motions must be accounted for. Coupling the rotor and fuselage introduces additional terms to the isolated rotor blade's equations of motion. Specifically, one new term introduced is the pitch effect on the blade's rotation; i.e., there is a "rocking" motion of the blade due to the body's pitching motion. In addition, changes in the helicopter's velocity and angle of attack influence the aerodynamic forces acting on the helicopter blade through the advance ratio and inflow ratio. The blade's flap and lag motions also couple with the body's force and moment equations, and these terms must be accounted for. Finally, the effects of reversed flow and compressibility are considered.

The equations of motion of a four-bladed coupled rotor/fuselage system with control can be expressed in the form

$$x'(\psi) = A(\psi)x(\psi) + B(\psi)u(\psi) \tag{1}$$

where the prime denotes differentiation with respect to the blade's azimuth ψ. Ignoring lateral motions, the state vector $x(\psi)$ is a 20-term vector whose first 16 elements represent the flap and lag angles and rates of each of the four rotor blades. The last four components consist of terms representing the helicopter's longitudinal motion, namely, forward velocity, angle of attack, body pitch angle, and body pitch rate. The control vector $u(\psi)$ is a four-term vector whose components represent collective, cyclic sine, and cyclic cosine blade pitch, and a rigid-body pitch control. The coupled rotor/fuselage equations reduce to the equations for the blades alone and the helicopter's airframe (excluding the rotor) alone when the coupling terms are ignored.

To verify the equations of motion, parameters representative of a typical helicopter with a single main rotor, the Rotor Systems Research Aircraft (RSRA), are used.[22-28] For this analysis a few of the main rotor's parameters have been altered so that unstable lag roots are generated for the uncontrolled system. Specifically, the main rotor radius was changed from 9.45 to 8.00 m, the main chord was changed from 0.648 to 0.300 m, and the blade's lagging natural frequency was changed from 0.500 to 0.100 rad/s. It is possible that these alterations may render the model to be unrepresentative of realistic helicopters; for this study, however, it was determined that, in order to demonstrate periodic control, the rotor parameters had to be altered to generate unstable lag roots. The helicopter's airframe (including the horizontal tail but excluding the main rotor) is modeled as a constant coefficient system; hence, standard feedback control may be used.[29] A control system using pitch feedback control to the main rotor's longitudinal cyclic pitch was designed to control the airframe's unstable rigid-body motion for forward velocities up to 175 m/s. Since this paper is examining the modal control technique for time-periodic systems, the coupled rotor/fuselage system roots generated with pitch feedback control are referred to as the uncontrolled roots.

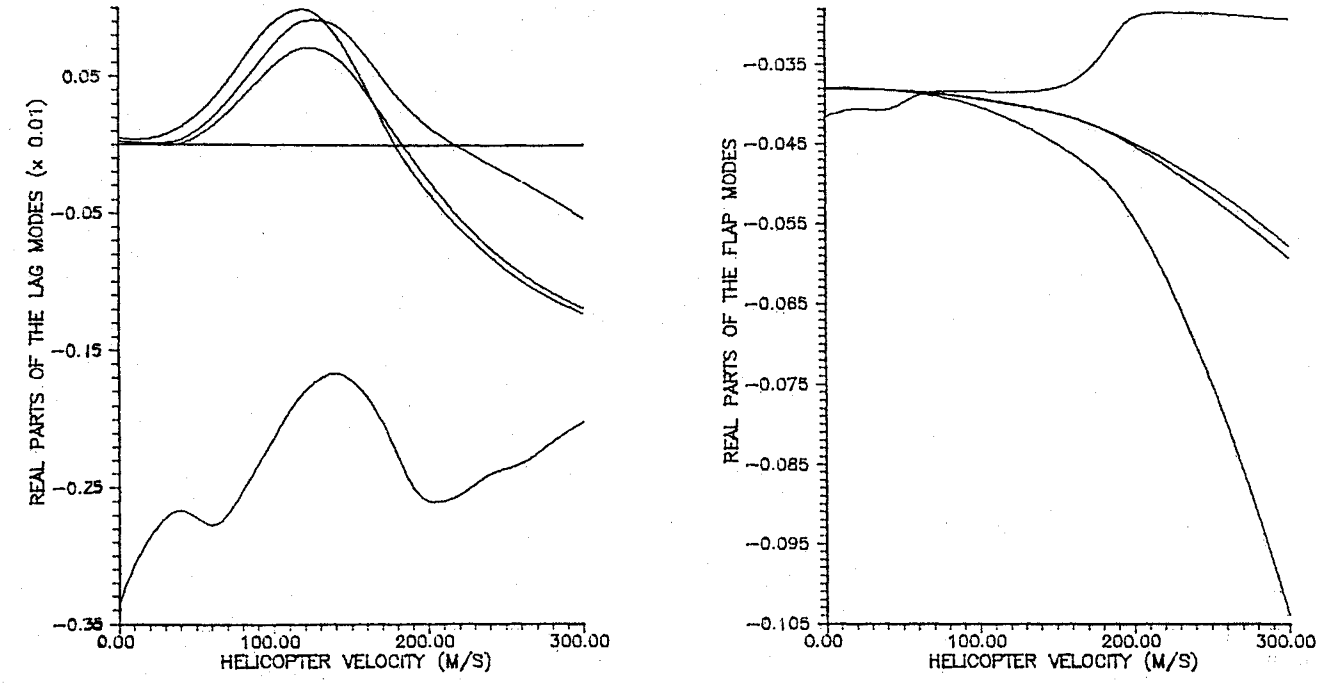
Figures 1a and 1b show the real parts of the uncontrolled lag and flap roots as a function of velocity. Note, in the figures, that the velocity ranges from hover to 300 m/s. It is doubtful that the rotor dynamic model is totally valid at the upper velocity ranges; however, such high speeds are used simply for the purpose of examining the effectiveness of the modal control technique throughout a wide range of velocities. It is understood that issues such as controllability, observability, and model validity at high speeds must be addressed if the modal control technique is to be developed as a practical concept.

In Fig. 1a, the magnitudes of the imaginary parts of the lag roots are approximately 0.100 from hover through 300 m/s. Note that, without control, six of the eight roots are unstable up to 218 m/s. Figure 1b plots the real parts of the flap roots as a function of velocity, and Fig. 2 shows the velocity root loci of the eight flap roots. The dashed and solid lines in Fig. 2 represents separate pairs of flap roots. From Figs. 1b and 2, it is clear that the flap roots are stable throughout the velocity regime inspected. Figure 3 shows the uncontrolled rigid-body roots with changing speeds; Fig. 3a is a velocity root locus of the roots, and Fig. 3b shows the real parts of the body roots as a function of velocity. The root representing the helicopter's vertical motion is shown to become unstable at speeds above 175 m/s.

Control

Theory

A control system, using the modal control technique described by Calico and Wiesel,[19,30] is designed to stabilize the



a) Lag roots

b) Flap roots

Fig. 1 Uncontrolled flap and lag roots.

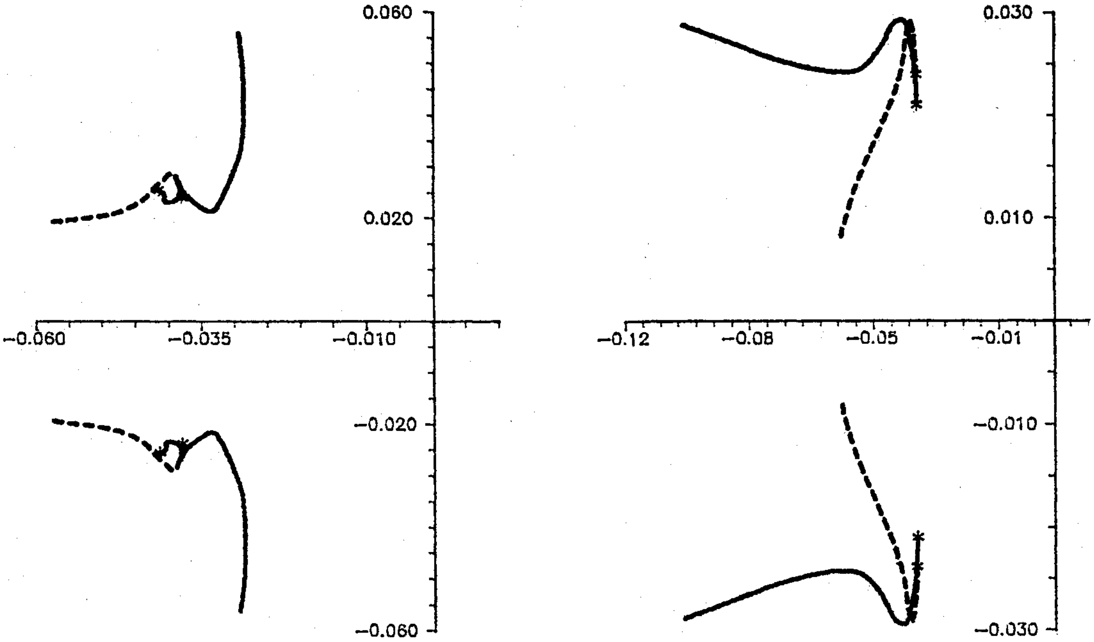


Fig. 2 Velocity root loci of uncontrolled flap roots.

linear periodic system

$$x'(\psi) = A(\psi)x(\psi) + B(\psi)u(\psi) \tag{2}$$

The control techniques developed in that work are briefly reviewed. Here $B(\psi)$ is the control matrix, and u is the vector of control inputs. The state transition matrix $\Phi(\psi,0)$ for the system of Eq. (2) may be expressed in the form

$$\Phi(\psi,0) = F(\psi)e^{J\psi}F^{-1}(0) \tag{3}$$

The matrix J is a constant matrix, and when put in Jordan normal form, its diagonal entries ω_i are termed Poincaré exponents. The matrix $F(\psi)$ is periodic with period 2π as are the coefficients of the original system [Eq. (2)]. Since any solution to Eq. (2) may be written in the form

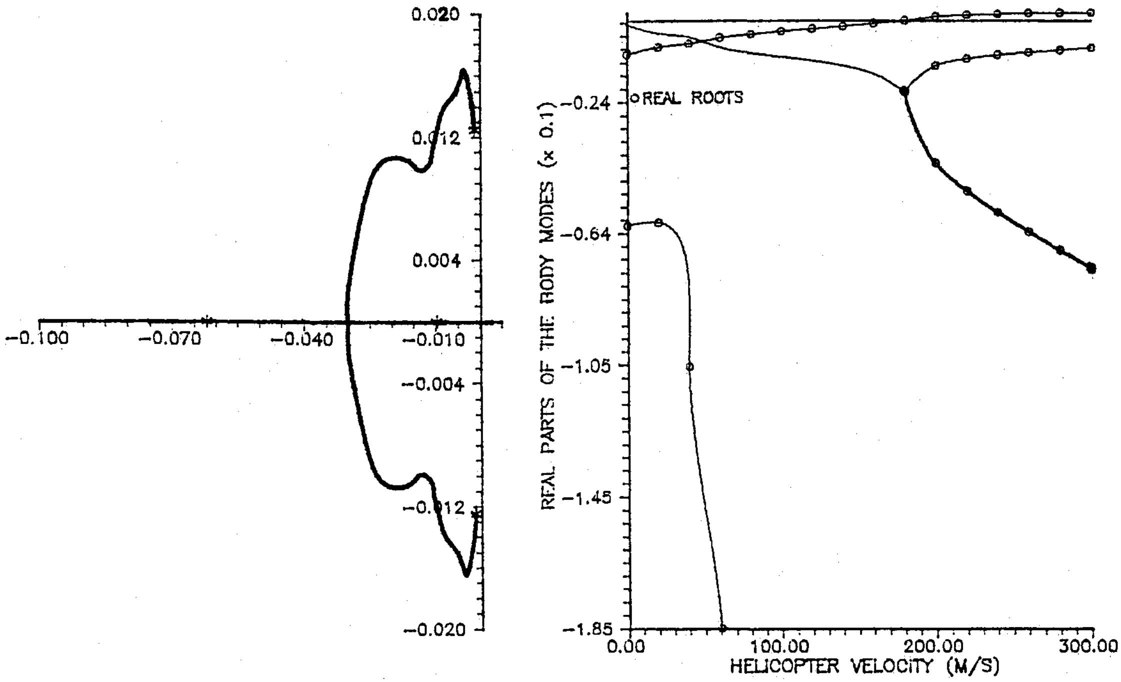
$$x(\psi) = \Phi(\psi,0)x(0) \tag{4}$$

the stability of any solution is determined by the Poincaré exponents ω_i. Modal control techniques were developed in Ref. 30 to modify the system Poincaré exponents. Both scalar and vector controllers were developed which adjusted one or more of the Poincaré exponents.

A single real Poincaré exponent ω_i for the system [Eq. (2)] may be changed to a value ω_i' while all other system exponents are unaltered by means of a scalar control of the form

$$u = k(\psi)f_i^{-T}(\psi)x(\psi) \tag{5}$$

where $f_i^{-T}(\psi)$ is the ith row of the $F^{-1}(\psi)$ matrix and $k(\psi)$ is a periodic scalar. With this control, the new exponent is



a) Velocity root locus

b) Roots vs velocity

Fig. 3 Uncontrolled body roots.

given by

$$\omega_i' = \omega_i + \left[f_i^{-T}(\psi)B(\psi)k(\psi)\right]_0 \tag{6}$$

where the subscript 0 refers to the constant term in the Fourier series of the bracketed term.

The scalar control given in Eq. (5) is not capable of moving, in a predictable fashion, two or more exponents. This capability is, however, vital if we are to successfully change the characteristic of a system with complex conjugate exponents. Although setting two exponents to specified new values is not presently possible, a technique is developed in Ref. 30 which does allow the sum of two exponents to be assigned a specific value. In the case where the roots are complex conjugates, this allows for assigning the real part of the complex conjugate pair. Noting by ω_i and ω_j the exponents to be changed and by ω_i' and ω_j' the new exponents, the scalar control

$$u = \left[k_i(\psi)f_i^{-T}(\psi) + k_j(\psi)f_j^{-T}(\psi)\right]x(\psi) \tag{7}$$

yields the results

$$\omega_i' + \omega_j' = \omega_i + \omega_j + \left[f_i^{-T}B(\psi)k_i(\psi) + f_j^{-T}B(\psi)k_j(\psi)\right]_0 \tag{8}$$

The gains $k_i(\psi)$ and $k_j(\psi)$ are scalar constants chosen so as to yield the appropriate constant value of the bracketed term in Eq. (8). The results shown in Eq. (8) are easily extended to setting the sum of an arbitrary number of exponents. The resulting relationship is that the sum of the new exponents is equal to the sum of the old exponents plus the constant value of the sum of the product of the individual modal gains times the corresponding f^{-T} term. This result, however, does not say anything about the individual exponents and for more than two modes it is difficult to find gains that stabilize each individual exponent.

A procedure is developed in Ref. 30 which allows for the exact placement of multiple exponents. The procedure, however, requires the use of multiple control inputs equal to the number of exponents to be assigned. For the case of moving two exponents with two control inputs, the control takes the form

$$u = \begin{bmatrix} k_{11}(\psi) & k_{12}(\psi) \\ k_{21}(\psi) & k_{22}(\psi) \end{bmatrix} \begin{bmatrix} f_1^{-T} \\ \hline f_2^{-T} \end{bmatrix} x \tag{9}$$

Here the subscripts 1 and 2 do not necessarily represent the first two exponents of a particular system but rather any two that we wish to change.

The gains $k_{ij}(\psi)$ necessary to assign to a pair of complex conjugate exponents ω_1 and ω_1^* new real values ω' and ω' are

$$k_{11}(\psi)g_{21}(\psi) + k_{21}(\psi)g_{22}(\psi) = \mathrm{Im}(\omega_1) \tag{10a}$$

$$k_{12}(\psi)g_{11}(\psi) + k_{22}(\psi)g_{12}(\psi) = \mathrm{Im}(\omega_1) \tag{10b}$$

$$k_{11}(\psi)g_{11}(\psi) + k_{21}(\psi)g_{12}(\psi) = -\omega_1' - \mathrm{Re}(\omega_1) \tag{10c}$$

$$k_{12}(\psi)g_{21}(\psi) + k_{22}(\psi)g_{22}(\psi) = -\omega_2' - \mathrm{Re}(\omega_1) \tag{10d}$$

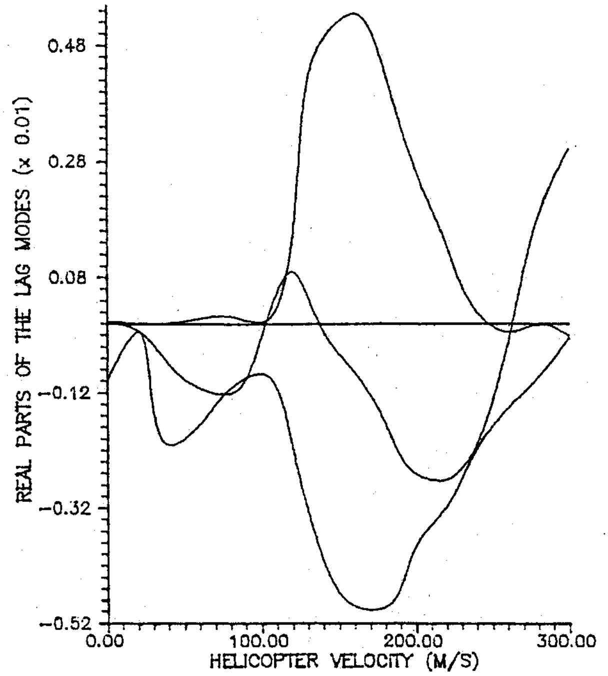
where the functions $g_{11},\ldots,g_{22}$ are the elements of the matrix $G(\psi)$ given by

$$G = \begin{bmatrix} f_1^{-T} \\ \hline f_2^{-T} \end{bmatrix} B(\psi) \tag{11}$$

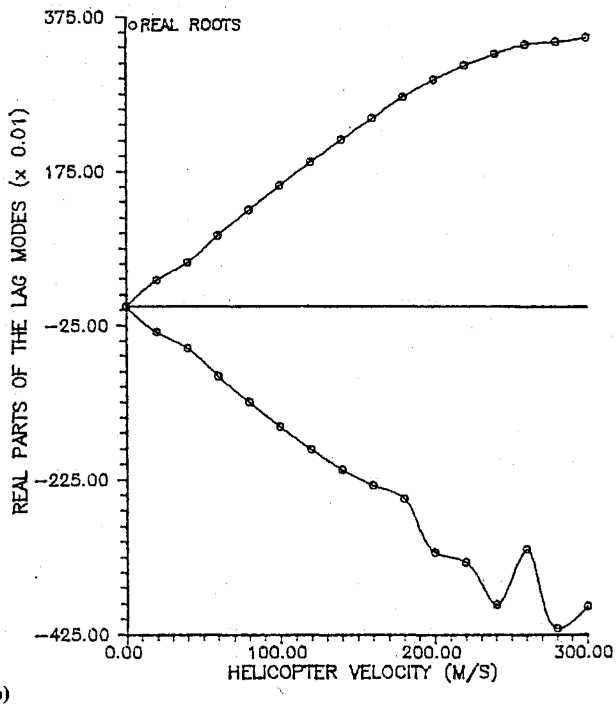
Similar results can be obtained for moving more than two modes with additional control inputs. It must be emphasized that the modal control technique shifts only the real parts of the unstable modes; there is no control over the imaginary parts. Thus, some closed-loop modes can be lightly damped, which could lead to some difficulties in actual implementation of the control technique.

Example

A scalar control system using the collective pitch was implemented using the results of Eq. (7). This control was designed to stabilize two of six unstable lag roots at a design point representing hover. The design point was chosen to be at hover since it was desired to apply the modal control technique at one point and observe the effectiveness of the control at the widest possible range of off-design conditions. The two most unstable lag roots were chosen as the pair to control. These roots have Poincaré exponents at $5.19259\times10^{-4}\pm1.05750\times10^{-1}i$, and the scalar collective pitch controller is designed to shift these roots so that the real parts of the new Poincaré exponents are -0.001. Figures 4a and 4b show the results of this control on the eight lag roots as a function of velocity. These figures show that at the design condition the two controlled lag roots have



a)



b)

Fig. 4 Lag roots as a function of velocity with control.

taken on the desired real parts and that the remaining six have not been altered.

However, when scalar collective pitch control is applied, the other three pairs of lag roots are significantly altered in off-design cases. This is readily apparent when comparing Fig. 1a to Fig. 4. The latter figure indicates that the lag roots for which the control was designed remains stable through 300 m/s. Another pair of roots is stabilized at lower speeds; however, they are unstable at higher speeds. A third pair of lag roots remains unstable throughout most of the velocity regime inspected; they are slightly stable at low speeds and at very high speeds. The final pair of roots, shown in Fig. 4b, splits off as two real Poincaré exponents just above the design point. One Poincaré exponent becomes unstable as the velocity increases while the other becomes more stable with increasing velocity. Except for the two lag roots that become real, the imaginary parts of the lag roots do not vary much as the velocity is increased; hence, velocity root loci are not plotted.

The collective pitch controller also alters the flap roots in off-design cases, as shown in Fig. 5. Note that Fig. 5a is a plot of the real parts of the flap roots as a function of velocity, and Figs. 5b and 5c plot the magnitudes of the imaginary parts of the flap roots vs velocity. Comparing these figures to Figs. 1b and 2 indicates that although the flap roots are unaltered at the design point, they do change as the velocity increases. Still, all four pairs of Poincaré exponents remain stable through 300 m/s. Note that one pair splits off as two real roots at higher velocities.

Finally, Fig. 6 shows the rigid-body roots as a function of velocity when the control is in place. Figure 6a plots the real parts of the body roots as the velocity is increased, and Fig. 6b plots the magnitude of the imaginary part of the oscillatory body roots as a function of velocity. The figures show that at the design point the collective pitch control does not alter these roots. Comparing this figure to Fig. 3, which shows the body roots as a function of velocity without control, indicates the movement of the rigid-body roots due to control. The oscillatory roots shown in Fig. 3 become more stable with increasing speeds; at higher velocities the roots become real, but remain stable. Although the pitch root is relatively unaffected by control, the vertical root becomes more stable when the control is applied. With just pitch attitude feedback control, this root becomes unstable at speeds above 175 m/s; when collective pitch control is applied this root becomes more stable with increasing speed (see Fig. 6).

Several other scalar control laws were developed which used either the cyclic sine or cyclic cosine blade pitch control or the body pitch control in place of the collective pitch. All yielded similar results in stabilzing pairs of roots. The vector control technique was also used successfully to control pairs of unstable lag roots.

Although modal control over a pair of unstable roots has been demonstrated, the system considered has six unstable lag modes for velocities below 218 m/s. We therefore seek to extend the above results to stabilization of the entire system. Scalar collective pitch control of four unstable lag roots was first attempted at various design points using the scalar control to set the sum of the real parts of the four roots. Unfortunately, gain values could not be found which shifted all four lag roots to stable locations. The same results were repeated when the other scalar and vector controllers were used. Even though the sum of the four roots was negative, finding values for the components of the gain matrix which also make each of the four roots stable is quite difficult once there are more than two unstable roots to control.

It is possible to stabilize more than two roots using the following technique. A modal control system that uses either a scalar or vector controller can be used to shift a pair of unstable roots to their desired locations at a specific design point. The resulting closed-loop system is a new linear system with periodic coefficients. Hence, another modal controller can be designed for this new system to shift a second pair of unstable roots to their desired locations. This process may be repeated until all of the instabilities are eliminated at the design point.

The technique described above is used to control the six unstable lag roots of the coupled rotor/fuselage system, and the results are summarized in Tables 1 and 2. Again, a scalar collective pitch controller is used. The design point is chosen to be at hover, and the real parts of the desired Poincaré exponents are chosen to be at -0.001. The Poincaré exponents of the uncontrolled system (in hover) are listed in the first column of Table 1 in the following order: the first four pairs of values represent the coupled lag roots; the next four pairs correspond to the coupled flap roots; and the remaining four Poincaré exponents (two real, one pair of complex conjugate roots) represent the coupled rigid-body roots.

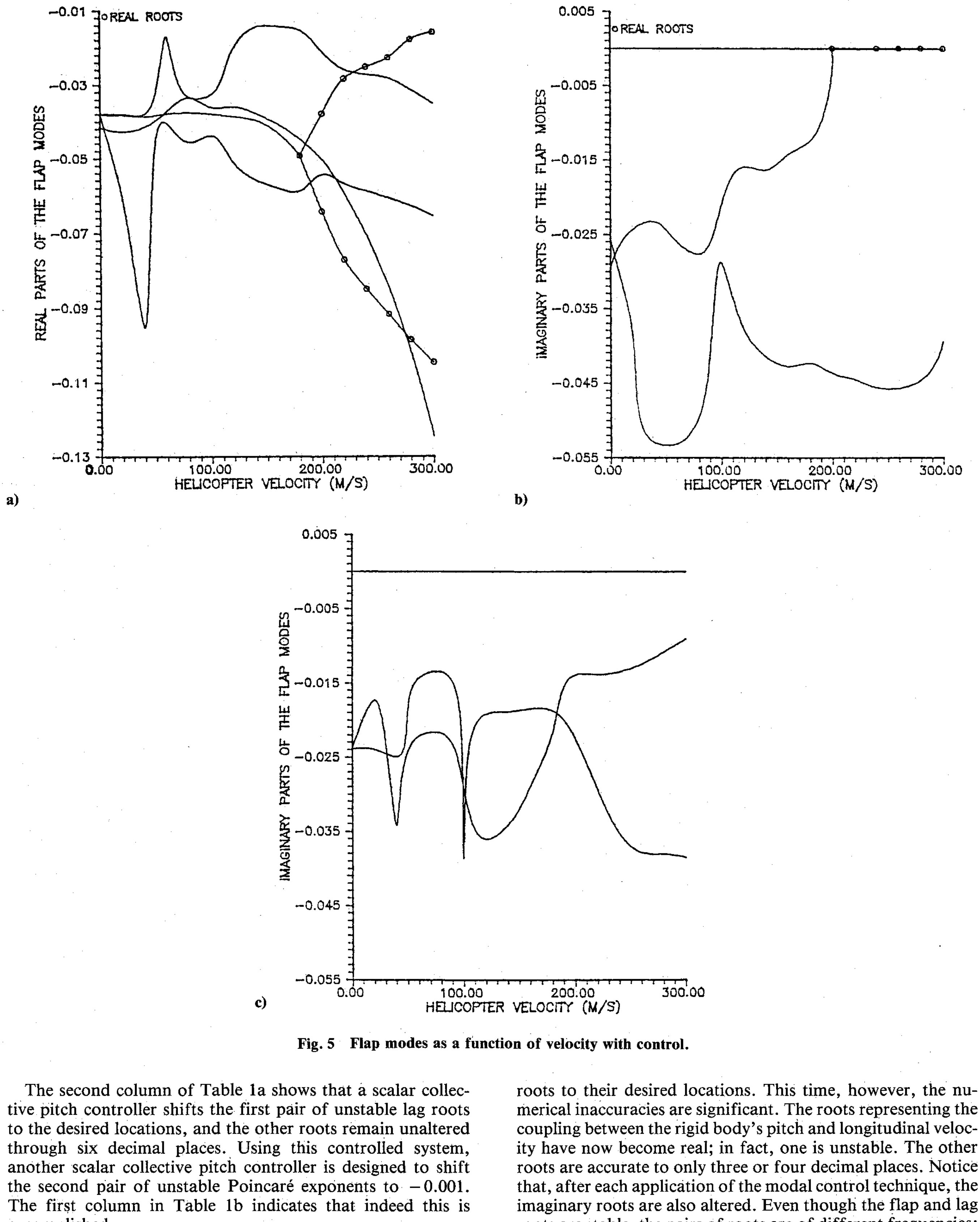


Fig. 5 Flap modes as a function of velocity with control.

The second column of Table 1a shows that a scalar collective pitch controller shifts the first pair of unstable lag roots to the desired locations, and the other roots remain unaltered through six decimal places. Using this controlled system, another scalar collective pitch controller is designed to shift the second pair of unstable Poincaré exponents to -0.001. The first column in Table 1b indicates that indeed this is accomplished.

Unfortunately, numerical inaccuracies can be noticed in the uncontrolled roots. The roots representing the coupling between the rigid body's pitch and longitudinal velocity (the next to the last pair of roots shown in the column) are different. In addition, the other roots are accurate to only four or five decimal places. The last column in Table 1b shows the results of controlling the final pair of unstable Poincaré exponents. Again, the scalar collective pitch controller shifts the unstable roots to their desired locations. This time, however, the numerical inaccuracies are significant. The roots representing the coupling between the rigid body's pitch and longitudinal velocity have now become real; in fact, one is unstable. The other roots are accurate to only three or four decimal places. Notice that, after each application of the modal control technique, the imaginary roots are also altered. Even though the flap and lag roots are stable, the pairs of roots are of different frequencies; hence, the physical symmetry of the helicopter is destroyed and, consequently, control would be quite difficult.

Table 2 shows a comparison between the gains required for each of the three scalar collective pitch controllers. Notice that the gains are all on the same order of magnitude.

The results summarized in Tables 1a and 1b indicate that the technique used to control multiple-blade instabilities works but that numerical inaccuracies are a factor when generating

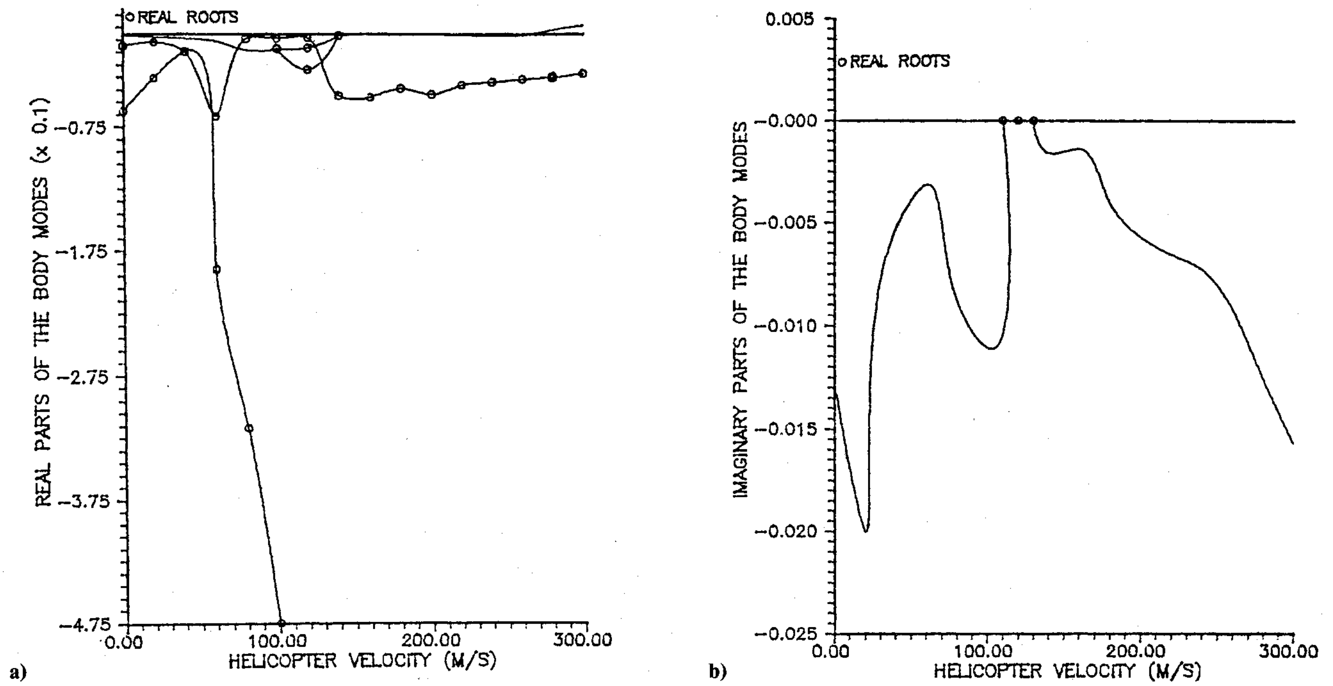


Fig. 6 Body roots as a function of velocity with control.

Table 1a Coupled rotor/fuselage Poincaré exponents

Uncontrolled system	1 Controller
$5.19259e^{-5} \pm 1.05749e^{-1}i$	$-9.99507e^{-4} \pm 3.02629e^{-1}i$
$3.05994e^{-5} \pm 1.05755e^{-1}i$	$3.05985e^{-5} \pm 1.05755e^{-1}i$
$9.04271e^{-6} \pm 1.05753e^{-1}i$	$9.05790e^{-6} \pm 1.05753e^{-1}i$
$-3.40809e^{-3} \pm 1.09155e^{-1}i$	$-3.40813e^{-3} \pm 1.09155e^{-1}i$
$-3.79558e^{-2} \pm 2.09968e^{-2}i$	$-3.79559e^{-2} \pm 2.09965e^{-2}i$
$-3.80523e^{-2} \pm 2.38908e^{-2}i$	$-3.80522e^{-2} \pm 2.38911e^{-2}i$
$-3.80693e^{-2} \pm 2.39050e^{-2}i$	$-3.80691e^{-2} \pm 2.39047e^{-2}i$
$-4.14992e^{-2} \pm 2.55519e^{-2}i$	$-4.14999e^{-2} \pm 2.55518e^{-2}i$
$-1.10548e^{-3} \pm 1.24843e^{-2}i$	$-1.10496e^{-3} \pm 1.24849e^{-2}i$
$-9.98093e^{-3}; -6.22026e^{-2}$	$-9.98096e^{-3}; -6.22036e^{-2}$

Table 1b Coupled rotor/fuselage Poincaré exponents

2 Controllers	3 Controllers
$-9.95073e^{-4} \pm 3.02579e^{-1}i$	$-9.92752e^{-4} \pm 3.02567e^{-1}i$
$-1.00400e^{-3} \pm 4.39454e^{-2}i$	$-1.37845e^{-3} \pm 6.52571e^{-2}i$
$9.23446e^{-6} \pm 1.05753e^{-1}i$	$-1.00003e^{-3} \pm 1.26413e^{-2}i$
$-3.43867e^{-3} \pm 1.09195e^{-1}i$	$-3.43899e^{-3} \pm 1.09193e^{-1}i$
$-3.80273e^{-2} \pm 2.11132e^{-2}i$	$-3.80132e^{-2} \pm 2.10375e^{-2}i$
$-3.80306e^{-2} \pm 2.38722e^{-2}i$	$-3.80480e^{-2} \pm 2.38974e^{-2}i$
$-3.80749e^{-2} \pm 2.38907e^{-2}i$	$-3.80662e^{-2} \pm 2.38986e^{-2}i$
$-4.16872e^{-2} \pm 2.55760e^{-2}i$	$-4.15845e^{-2} \pm 2.54493e^{-2}i$
$-5.53107e^{-4} \pm 1.32186e^{-2}i$	$7.46453e^{-1}; -7.43286e^{-1}$
$-1.00072e^{-2}; -6.23713e^{-2}$	$-9.97911e^{-3}; -6.22442e^{-2}$

Table 2 Controller gains

1 Controller	2 Controllers	3 Controllers
-20.1367	-20.1367	-20.1367
9.8640	9.8640	9.8640
0.0000	25.9870	25.9870
0.0000	21.0596	21.0596
0.0000	0.0000	22.2350
0.0000	0.0000	5.4816

the uncontrolled roots, and become noticeable when the modal control technique is used more than once at a particular design point.

As a final note, the modal control technique can also be used to stabilize the main rotor's blades with individual blade controllers. As Calico and Wiesel observed, any number of unstable blade roots can be controlled with simple one-blade scalar controllers.[19,30]

Conclusions

The coupled rotor/fuselage system was used to analyze the modal control technique developed by Calico and Wiesel.[19,30] Even though the system was of order 20, there were no numerical difficulties in designing a modal controller to place a single pair of unstable roots. It was shown that, at a specific design point, scalar and vector control shifted a pair of unstable Poincaré exponents to desired locations without altering the other roots in the system. In addition, the controllers, designed at a single point, stabilized the controlled lag roots over a wide range of off-design cases. However, the controllers altered the other roots in off-design cases.

Finding gains that stabilized more than two roots proved to be very difficult. However, the modal control technique was used to eliminate multiple-blade instabilities by first controlling a pair of unstable roots at a specific design point. The resulting closed-loop system was a new linear system with periodic coefficients. Another modal controller was designed for this new system to shift a second pair of unstable roots to desired locations. This process was repeated until all instabilities were eliminated. Numerical inaccuracies, unfortunately, became noticeable when modal control was used more than once.

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